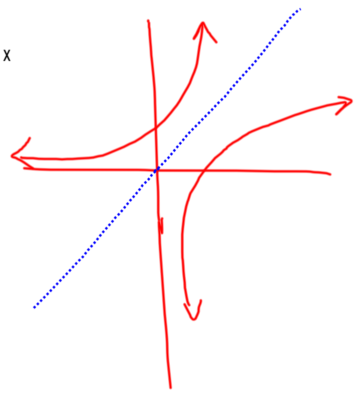
10-2 Logarithms and Logarithmic Functions

$y = 2^x$

0, 1
1, 2
2, 4

4, 2
2, 1
1, 0



Inverse of $y=2^x$

Does this graph have an inverse?

$y = 2^x$

$x = 2^y$

$y = 2^x$ $y = \log_2 x$ "y = log base 2 of x"

Inverses of each other

$y = b^x$ $y = \log_b x$

What is the inverse?

$y = 10^x$

2, 100
3, 1000

$y = \log_{10} x$

$\log_{10} 100 = 2$
$\log_{10} 1000 = 3$

Characteristics of a Logarithmic Function

1. The function is continuous and one-to-one.
2. The domain is the set of all positive real numbers.
3. The y-axis is an asymptote of the graph.
4. The range is the set of all real numbers.
5. The graph contains the point (1, 0). That is, the x-intercept is 1.

Suppose b and x are positive, and b $\neq 1$, then, there is a number y such that:

$$\log_b x = y \text{ iff } b^y = x$$

(Used to convert between logarithmic and exponential form.)

$\log_{10} 1000 = 3$ $10^3 = 1000$

$$\log_b x = y \text{ iff } b^y = x$$

Logarithmic Form	Exponential Form
$\log_2 16 = 4$	$2^4 = 16$
$\log_2 8 = 3$	$2^3 = 8$
$\log_2 1 = 0$	$2^0 = 1$
$\log_2 x = y$	$2^y = x$
$\log_b x = y$	

$$\log_b x = y \text{ iff } b^y = x$$

Logarithmic Form	Exponential Form
~~$\log_{10} 1000 = 3$~~	
$\log_{16} 4 = .5$	$16^{\frac{1}{2}} = 4$
$\log_3 27 = 3$	$3^3 = 27$
$\log_9 81 = 2$	$9^2 = 81$
$\log_5 125 = 3$	$5^3 = 125$

Evaluate a logarithmic expression.

ex
$\log_2 64 = y$

$2^y = 64$

2^6

6

$\log_2 2^6$

6

ex
$\log_{25} 5 = \frac{1}{2}$

$\log_{25} 5 = y$

$25^y = 5$

$5^{2y} = 5$

$2y = 1$

$y = \frac{1}{2}$

ex
$\log_{10} 0.1$

$\log_{10} \frac{1}{10} = -1$

Remember:
Two functions are inverses iff
$[f \circ g]\, x = x$ and $[g \circ f]\, x = x$

$[f \circ g](x) = b^{\log_b x} = x$

$[g \circ f](x) = \log_b b^x = x$

Inverses of each other

$y = b^x \quad y = \log_b x$

$f(x) = b^x \quad g(x) = \log_b x$

$b^{\log_b x} = x$

Properties of logs

$\log_b b^x = x$

ex
$\log_2 2^5 = 5$

ex
$\log_4 4^2 = 2$

ex
$\log_7 49 = 2$

ex
$3^{\log_3 5}$

5

ex
$8^{\log_8 10}$

10

DO:

1. $\log_{1/2} 32$
2. $\log_9 27$
3. $\log_5 125$
4. $\log_8 4$
5. $9^{\log_9 5}$
6. $\log_{\sqrt{3}} 9\sqrt{3}$

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